

Lesson 2-4 Rates of Change and Tangent Lines

*****HOMEWORK ANSWERS*****

Lesson 2-4 HW;

pp. 93-95 / 1, 2, 7, 10, 11, 15, 33, 36, 44

1. $f(x) = x^3 + 1$

a. $A_{ROC} = \frac{f(3) - f(2)}{3 - 2} = \frac{28 - 9}{1} = \boxed{19}$

b. $A_{ROC} = \frac{f(1) - f(-1)}{1 - (-1)} = \frac{2 - 0}{1} = \boxed{2}$

2. $f(x) = \sqrt{4x+1}$

a. $A_{ROC} = \frac{f(2) - f(0)}{2 - 0} = \frac{3 - 1}{2} = \boxed{1}$

b. $A_{ROC} = \frac{f(12) - f(10)}{12 - 10} = \frac{7 - \sqrt{41}}{2} \approx \boxed{.298}$

7. Using:

a. $Q_1 = (10, 225)$ $Q_2 = (14, 375)$ $Q_3 = (16.5, 475)$

$Q_4 = (18, 550)$ $P = (20, 650)$

Secant	Slope (m/sec)
PQ ₁	43
PQ ₂	46
PQ ₃	50
PQ ₄	50

b. $\approx 50 \text{ m/sec}$

10. $y = x^2 - 4x$ at $x = 1$

a. $\lim_{h \rightarrow 0} \frac{(x+h)^2 - 4(x+h) - (x^2 - 4x)}{h}$

$= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 4x - 4h - x^2 + 4x}{h}$

$= \lim_{h \rightarrow 0} \frac{h^2 + 2xh - 4h}{h}$

$= \lim_{h \rightarrow 0} (h + 2x - 4) = \boxed{2x - 4}$

at $x = 1$
 $(2x) - 4 = \boxed{-2}$

b. $f(1) = 1^2 - 4(1) = -3$
 $f(1) = -3$

$\boxed{y + 3 = -2(x - 1)}$ T.L.

C. $\boxed{y + 3 = \frac{1}{2}(x - 1)}$ N.L.

Lesson 2-4 Rates of Change and Tangent Lines

Lesson 2-4

11. $y = \frac{1}{x-1}$ at $x=2$

a. $\lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)-1} - \frac{1}{x-1}}{h} =$

$\lim_{h \rightarrow 0} \frac{x-1 - (x+h-1)}{(x+h-1)(x-1)h} =$

$\lim_{h \rightarrow 0} \frac{-h}{h(x+h-1)(x-1)} =$

$\lim_{h \rightarrow 0} \frac{-1}{(x+h-1)(x-1)} =$

$= \frac{-1}{(x-1)^2}$

When $x=2$, $m = -1$

b. $f(2) = 1$

$y - 1 = -1(x - 2)$ T.L.

c. $y - 1 = 1(x - 2)$ N.L.

15. $f(x) = \begin{cases} 2-2x-x^2, & x < 0 \\ 2x+2, & x \geq 0 \end{cases}$ at $x=0$

$\lim_{h \rightarrow 0^-} \frac{2-2(x+h)-(x+h)^2 - (2-2x-x^2)}{h} =$

$\lim_{h \rightarrow 0^-} \frac{-2h - 2xh - h^2}{h} =$

$\lim_{h \rightarrow 0^-} \frac{-2h - 2xh - h^2}{h} =$

$\lim_{h \rightarrow 0^-} \frac{-2-2x-h}{1} = -2-2x$

at $x=0$, $m = -2$
No; Limit of difference quotient DNE

$\lim_{h \rightarrow 0^+} \frac{2(x+h)+2 - (2x+2)}{h} =$

$\lim_{h \rightarrow 0^+} \frac{2x+2h+2 - 2x-2}{h} =$

$\lim_{h \rightarrow 0^+} \frac{2h}{h} = 2$

33.

$\lim_{h \rightarrow 0} \frac{(x+h)^2 + 4(x+h) - 1 - (x^2 + 4x - 1)}{h} =$

$\lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 4x + 4h - 1 - x^2 - 4x + 1}{h} =$

$\lim_{h \rightarrow 0} \frac{2xh + h^2 + 4h}{h} =$

$\lim_{h \rightarrow 0} \frac{h(2x+h+4)}{h} = 2x+4$

$2x+4=0$

$2x=-4$

$x=-2$

plus into original function

$f(-2) = (-2)^2 + 4(-2) - 1 = -5$

minimum of parabola

Lesson 2-4

$$36. \lim_{h \rightarrow 0} \frac{9 - (x+h)^2 - (9 - x^2)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{9 - x^2 - 2xh - h^2 - 9 + x^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-2xh - h^2}{h} = -2x \leftarrow \text{slope of T.L.}$$

★ $(1, 12)$ is not on the graph $y = 9 - x^2$
 So find eq. of T.L.s thru $(1, 12)$ and $(x, 9 - x^2)$

$$\frac{\Delta y}{\Delta x} = \frac{9 - x^2 - 12}{x - 1} = -2x$$

$$-2x(x-1) = -x^2 - 3$$

$$-2x^2 + 2x = -x^2 - 3$$

$$2x = x^2 - 3$$

$$0 = x^2 - 2x - 3$$

$$0 = (x-3)(x+1)$$

$$x=3 \quad x=-1$$

$$f(-1) = 9 - (-1)^2 = 8 \quad (-1, 8) \text{ point of tangency}$$

$$m_{\text{T.L.}} = -2(-1) = 2$$

$$\boxed{y - 8 = 2(x + 1)}$$

$$f(3) = 9 - 3^2 = 0 \quad (3, 0) \text{ point of tangency}$$

$$m_{\text{T.L.}} = -2(3) = -6$$

$$y - 0 = -6(x - 3)$$

$$\boxed{y = -6(x - 3)}$$

44. E.